Probabilistic analysis of a gravity dam in Norway

A new approach to dam safety in Norway?

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ABSTRACT: During the spring 2017, a probabilistic analysis of dam Reinoksvatn was carried out, in order to gain experience with this type of analysis and how it can be used to evaluate safety of existing dams.

The Reinoksvatn dam is a 20 m high and 400 m long concrete gravity dam located in Sørfold municipality in Norland County. The dam was built in the period 1985-86 and is connected to the Kobbelv hydro power plant.

The analysis has been carried out by applying two different approaches:

1. In the first part, a relatively simple element model has been used to calculate safety against failure based on so-called probabilistic methods.
2. In the second part, estimate the safety has been carried out using deterministic methods. In this part of the project, an element model has been used and provide a more accurate description of the actual behavior of the dam.

This analysis is part of a lager Norwegian Research and Development project, named “Dam safety in an overall perspective” that is administrated by EnergiNorge. This is a joint project with participants from the Norwegian dam safety sector. One of the objects of this project is to look at alternative approaches to evaluate the safety of existing concrete- and masonry dams.

1 Introduction

Requirements for stability of concrete dams in the current regulations are based on simplifications, which in many cases are conservative. As a consequence unnecessary rehabilitation works may be carried out on dams that are safe, but does not meet the safety requirements. It is therefore desirable to look into how the assumptions for the calculations affect stability, in order to provide a better model to assess the actual safety and capacity. In this respect, it is also desirable to identify which elements that have most effect on the uncertainty when calculating sliding and overturning of the dam.

In a probabilistic analysis, a safety factor is not relevant, instead a probability distributions for all variables is defined. Using statistical methods, one can thus calculate possible outcomes that combined with a failure criterion gives a probability of failure. This probability is then compared to a safety requirement. This method of analysis form the basis for the partial factors for materials and loads in e.g. the Eurocodes.

This probabilistic analyses is based on the “Probabilistic Model Code for Concrete Dams” written by Marie Westberg Wilde and Fredrik Johansson for Energiforsk in Sweden [1].In addition, data from the Joint Committee on Structural Safety (JCSS) are used [2]. This committee is supported by six international associations in construction engineering – CEB, CIB, FIB, IABSE and RILEM.

Analysis in this report is carried out on the Reinoksvatn dam, a 20 m high and 400 m long concrete gravity dam, to illustrate the issues above with a probabilistic analysis. Traditional calculation of dam stability shows that the dam has insufficient sliding capacity when friction angle of 45^o is assumed.

Figure 1: Airplane view and downstream side of the dam.

2 Dam stability

Uncertainty is present in loading, materials and models. The Norwegian regulations on dam safety (Damsikkerhetsforskriften) [3] handle this by require a minimum safety against sliding and overturning. This method is called a deterministic method. A deterministic method is based upon empirical data and/or calibrations with probabilistic methods.

The problem with deterministic methods is that they provide limited understanding of what causes uncertainties and how an increased reliability can be achieved. Deterministic methods are simple to apply in design, and therefore well suited for design of new structures. For reassessment of existing structures however, conservative assumptions can lead to extensive rehabilitations that are not really necessary.

2.1 Probabilistic design

In a probabilistic analysis, the uncertain variables are defined directly, and the probability of failure is calculated based on these variables. The method also returns how much each variable affect the reliability, and based on this it is possible to take effective actions to increase the reliability. These actions can range from doing measurements to get more accurate data (reducing uncertainty), to strengthening a specific component of the structure.

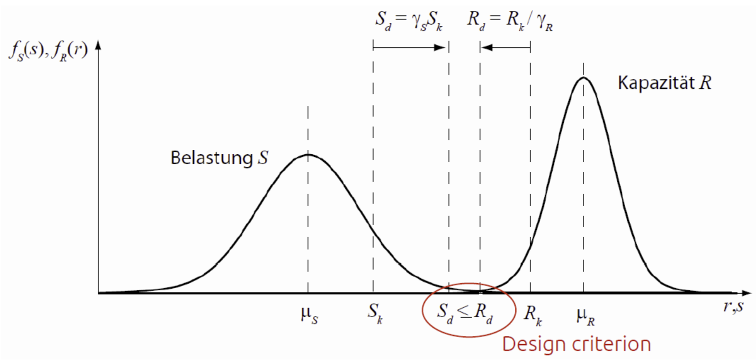

Figure 2: The overlapping region between load (Belastung) and resistance (Kapazität) define situation where the structure will fail [4]

Figure 2 illustrate how both the loading on a structure, and the resistance of the structure is uncertain. A deterministic method uses a load factor (γ_S) and a material factor (γ_R) which increases the characteristic load, and decreases the characteristic capacity. If the design load is less than the design resistance, the structure is assumed to have sufficient safety. These factors are based on an acceptable level of risk. A probabilistic method calculates the probability of failure directly and compares that to the acceptable level of risk.

Eurocode 0 allows the use of probabilistic design as an alternative to the use of partial safety factors in section 3.5 [5].

2.2 FEM-model

A 2D finite element model of a 1-meter wide section of the Reinoksvatn dam, is modeled and analyzed with the finite element program SOFiSTiK 2016 [6].

A set of variables is used to define the uncertain variables of the dam. A statistical module of SOFiSTiK called RELY is used to define these variables and evaluate how they influence the safety of the dam. The variables are defined in section 3, and include Self weight, Geometry and loading.

Two criterions are used to define failure of the dam. One criterion define sliding failure and the other define overturning. These are explained in detail in section 4.

The dam is modelled with 2D shell elements with a linear elastic material model. The interface towards the rock foundation is modelled with springs. Both a linear and a nonlinear model is used to model the concrete-rock interface. The nonlinear springs do not carry any tension loads, and lose their shear capacity when not in compression.

Two different approaches are used for evaluation of the probabilistic problem. A method called FORM (First Order Reliability Method) was primarily used to calculate the reliability. This method is based upon a linearization of the failure criterion, and is a very efficient way of calculating the reliability. In addition to returning the probability of failure for the structure, the method returns the sensitivities for each variable. However, as the method is based upon a linearization, it does not converge well for highly non-linear problems. An illustration of the FORM method is given in Figure 3.

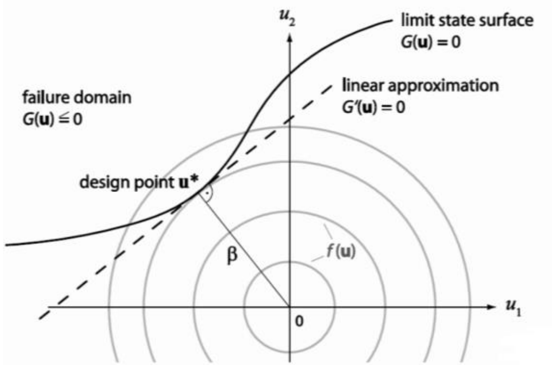

Figure 3: A visualization of the FORM method.

An alternative approach is to use a Monte Carlo simulation. This method is a level 3 method, or full probabilistic method, as it does not include any linearization. The method is considered exact and also works for highly non-linear problems, but is computationally costly. A large amount of realizations of the problem is carried out and the probability of failure is defined as the number of failed samples divided by the total number of samples.

In this project, we have mainly used the FORM method, and then used Monte Carlo with a limited number of simulations to verify the results.

Table 1: Relation between reliability index and probability of failure [5]

Pf	**10^{-1}**	**10^{-2}**	**10^{-3}**	**10^{-4}**	**10^{-5}**	**10^{-6}**	**10^{-7}**
β	1,28	2,32	3,09	3,72	4,27	4,75	5,20

3 Definition of variables

The variables describing the loads acting on the dam and the resistance of the structure is modeled with probability density functions (PDF). A common PDF for natural random variables is a normal distribution, described by a mean value and a standard deviation. Other distributions used in this project is log-normal distributions, which constrain the PDF to only positive values, and Gumbel distributions which models extreme values well.

The coefficient of variation is defined as the ratio of the standard deviation, σ, to the mean, μ.

$$c_v = \frac{\sigma}{\mu} \tag{3}$$

3.1 Stiffness of concrete

The E-modulus of the concrete is modelled with a normal distribution, with a mean value equal to the E-modulus of B35 (34 000 MPa) and a coefficient of variation (C.o.V) of 0.15. This C.o.V is recommended by the JCSS model code [2], table 3.1.1.

3.2 Structural self-weight

Structural self-weight is modelled with a normal distribution, with a mean value of 24 kN/m^3 and a coefficient of variation of 0.04. The C.o.V is recommended in table 2.1.1 in JCSS model code. [2]

In PMCD chapter III:.1.4 (Probabilistic model code for concrete dams) a reduction of the C.o.V of 0.85 is proposed. [1] This gives a C.o.V of 0.034.

3.3 Cohesion

The PMCD have no recommend values for cohesion. We have chosen to include this in our analysis to see what kind of effect cohesion has on the sliding capacity. The value is taken from tests carried out by NORUT (Northern Research Institute) at the Målset dam [7]. The probability density function is implemented in the analysis as a log-normal distribution with a mean value of 0.389 MPa and a coefficient of variation of 0.289.

The figure below shows a normal distribution, which was first implemented with the same values. This gave some problems due to the possibility of negative values for the cohesion, which is impossible to achieve in reality. A log-normal distribution was therefore considered to provide the best fit to the experimental data. This distribution is considered conservative as it is shifted more towards lower values of cohesion.

Eurocode 2 recommends a cohesion of $0.2 \cdot f_{ctd}$ for a smooth surface between concrete cast at different times. This would give a cohesion of 0.328 MPa for B30 concrete. A smooth surface is defined as a slipformed or extruded surface, or a surface left without further treatment after vibration. [8] This value is similar to the mean value proposed by NORUT.

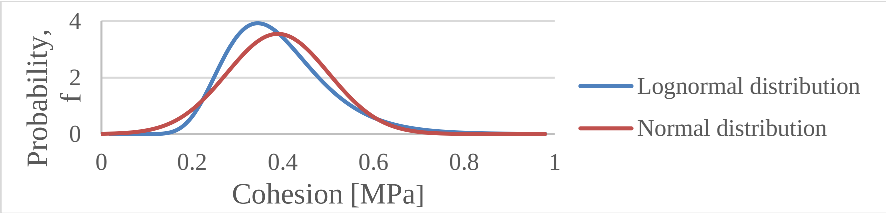

Figure 4: Probability density function, cohesion

3.4 Friction

The friction angle is included according to chapter III:.3 in PCCMD, with a tan φ-mean value of 35° (0,7) and a standard deviation of 1,75° (0,031). [1]

Eurocode 2 proposes a friction factor of 0.6 for a smooth surface between concrete cast at different times. This is somewhat lower than the mean value applied in the analysis, but the surface between rock and concrete is rougher than the smooth surface, defined in EC2. [8]

3.5 Rock bolts, yield strength of the reinforcement

The yield strength of rock bolts is modelled with a lognormal probability function. We have included this in the analysis only on the capacity side for sliding according to the shear formula in EC2. The mean value is 180 MPa, with a coefficient of variation of 0.89.

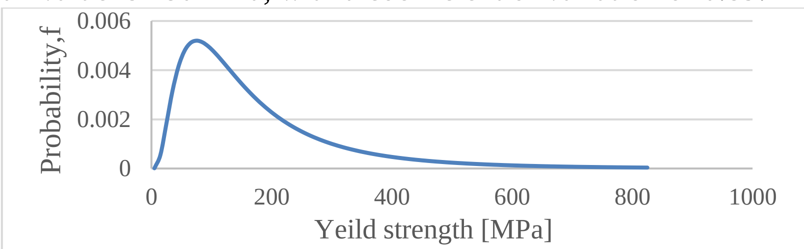

Figure 5: Probability density function, yield strength of reinforcement

3.6 Hydrostatic pressure

The probability density functions for the water level is based on daily measurements of the dam water level from 1988 to 2017. In the analysis, we have assumed two different load situations: (i) Winter season (November - May) including Ice loading and (ii) Summer season (June – October) which include flooding.

Figure 6 (graph to the left) shows the probability density function for the water level during winter, based on the flood histogram, where the maximum water level is taken as 19.5 m, 0.5 m above highest operating water level (HRV). The probability of the water exceeding 19.0 m is 1.7 % for this distribution. Figure 6 also shows the probability density function for the flood case (graph to the right), based on the flood histogram. The maximum water level is taken as 20 m (dam height). This density function gives probability of the water level exceeding 19.8 (1.5·Qdim) of 0.2%.

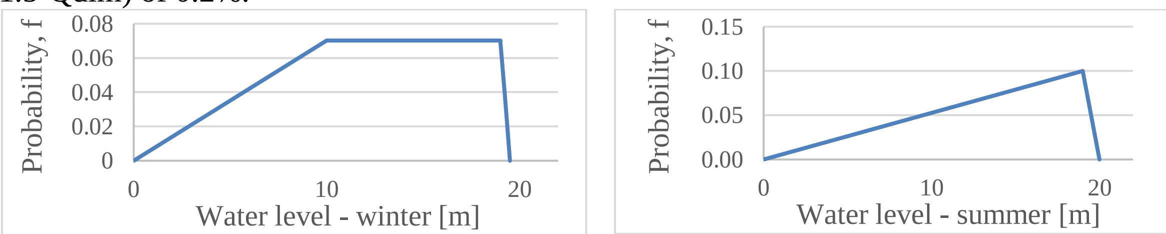

Figure 6: Probability density function for Water level (**Left**: Ice load case - **Right**: Flood case)

In probabilistic analysis, the yearly probability of the variable is used as reference. We chose to use the monthly maximum values to base our probability densities on, due to larger data sampling.

The uplift pressure is implemented in the calculations according to normal practice. It varies linearly from the heel to the toe of the dam if the resultant is within 1/3 of the base width. If the resultant is downstream this area, the uplift pressure is constant over the area without compression.

3.7 Ice loading

The ice load according to PMCD, should be described by a log-normal distribution, shown in the figure below. According to NVE guidelines, an alternative method for calculating ice load returns a load of 125 kN/m for Sørfold municipality. This is based on a frost level with a return period of 100 years.

Initially, the analysis was executed with the log-normal distribution proposed in PMCD which resulted in unrealistic high ice load over 800 kN/m, as the log-normal distribution ranges from 0 to ∞. As the ice load is in reality a deformation load, we implemented a trapezoidal distribution to fit the log-normal curve but not exceed 150 kN/m. This distribution gives a 3% annual probability of the ice load to exceed 125 kN/m.

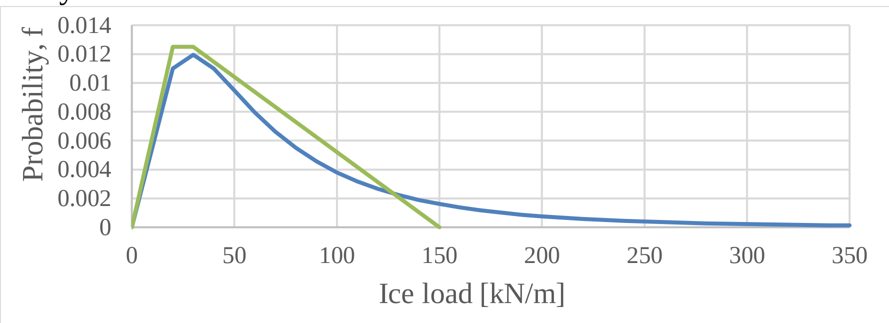

Figure 7: Probability density function, ice load [1]

The ice load is applied 0.25 meters below the water level, e.g. the ice thickness is assumed 0.5 meter with the resultant acting at the center of the layer of ice.

3.8 Geometric variation

The geometric variation is included as a delta applied to the height and width of the dam. This probability density function is based on normal building tolerances, to investigate what kind of impact this can have on failure of the dam. The probability density function is described with a normal distribution, with a mean value of 0 and a standard deviation of 0.1 m.

4 Failure criterion

A failure criterion has to be defined for each failure mode. Failure happens when the failure criterion is less than, or equal to, zero. To achieve convergence with FORM (chapter 2.2) the failure criterion must be expressed so that the software also know how close it is to failure. A normal expression is:

$$R - F > 0 \tag{4}$$

Where R is resistance and F is loading.

4.1 Overturning

The failure criterion is specified as $e_{Ed} - e_{Rd} > 0$

The eccentricity of the resultant from the dam toe, e_{Ed}, is calculated by reading the spring forces under the dam and finding the resultant placement based on the pressure distribution of these. Overturning occurs when the resultant is downstream of the dam toe (i.e. $e_{Ed}<0$). In reality crushing of the concrete at the dam toe will occur before this, so the design value is set to e_{RD} = B/24 (≈0.67m).

4.2 Sliding

The sliding capacity is based on the formula for design shear capacity for casting joints, according to Eurocode 2 [8]. This formula contains cohesion, friction and bolt capacity. This makes it possibel to assess the contribution of other variables in addition to friction. Assuming a plane surface between the rock and the dam, makes the model simple and transparent while providing results that are directly comparable to the requirements of existing regulations.

The failure criterion is specified as $V_{Rd} - V_{Ed} > 0$, where the sliding force, V_{Ed}, is the sum of all horizontal forces, while the design sliding resistance, V_{Rd}, is defined as:

$$V_{Rd} = N' \cdot \mu + A_c \cdot c + \mu \cdot f_y \cdot A_s \quad (5)$$

Where, **N'** = the sum of all vertical forces, **μ** = the friction coefficient, $\mathbf{A_c}$ = the area of the foundation in pressure, **c** = the cohesion, $\mathbf{f_y}$ = yield strength of rock bolts and $\mathbf{A_s}$ = the area of the rock bolts.

Cohesion, friction and rock bolts are only included in estimation of sliding capacity. Cohesion and friction does not affect the stability against overturning.

5 Results

The results of the probabilistic analysis gives us a β, which should exceed the target safety index for the dam consequence class, given in table PI-6.2 in PMCD [1]. It also returns the design point, which is the combination of variables leading to failure that is most likely to occur. The alpha values reflect the sensitivity of each variable and reflect how important that variable is compared to the other variables. To improve the safety of the dam the most sensitive variable should be addressed first.

The results are shown in the tables below.

Table 2: Probability of failure and safety index (β).

	Situation 1 Winter season with ice load		**Situation 2 Summer season with flooding**	
Failure mode	**Probability of failure**	**β – safety index**	**Probability of failure**	**β – safety index**
Overturning	$2.14 \cdot 10^{-7}$	5.05	$6.42 \cdot 10^{-8}$	5.28
Sliding	$5.17 \cdot 10^{-7}$	4.88	$2.93 \cdot 10^{-7}$	5.12

Table 3: Sensitivity values (Alpha) and values at the design point where failure is most likely to occur.

Variable	Situation 1 Winter season with ice load				Situation 2 Summer season with flooding			
	Overturning		Sliding		Overturning		Sliding	
	Failure value	Alpha	Failure value	Alpha	Failure value	Alpha	Failure value	Alpha
E-modulus [MPa]	34 000	0 %	34 000	0 %	34 000	0 %	34 000	0 %
Ice load [kN/m]	120.2	11 %	119.76	12 %	Not relevant		Not relevant	
Self-weight[kN/m^3]	20.90	56 %	21.10	53 %	20.50	66 %	20.67	63 %
Rock bolts [kPa]	Not included		129 358	0 %	Not included		129 358	0 %
Water level [m]	19.31	30 %	19.30	32 %	19.83	32 %	19.83	34 %
Delta height [m]	-0.06	1.5 %	-0.05	1.0 %	-0.07	1.7 %	-0.06	1.2 %
Delta width [m]	-0.03	0.4 %	-0.04	0.8 %	-0.03	0.4 %	-0.05	0.8 %
Cohesion [MPa]	Not relevant		0.32	1.1 %	Not relevant		0.32	0.9 %
Friction	Not relevant		0.70	0 %	Not relevant		0.70	0 %

5.1 Verification of results

The four cases with their corresponding failure load is analyzed with non-linear springs under the foundation to verify that the linear springs used in the probabilistic analysis is valid. In the probabilistic analysis the springs under the foundation is modeled as linear. Due to the huge number of analysis need to perform the probabilistic analysis, the linear spring reduces the time for analyzing and ensures convergence of the β–safety factor. The non-linear spring model include non-tension springs along the foundation that has no stiffness in tension.

The figure below show the forces and deformations in the springs for the failure point for sliding with ice load.

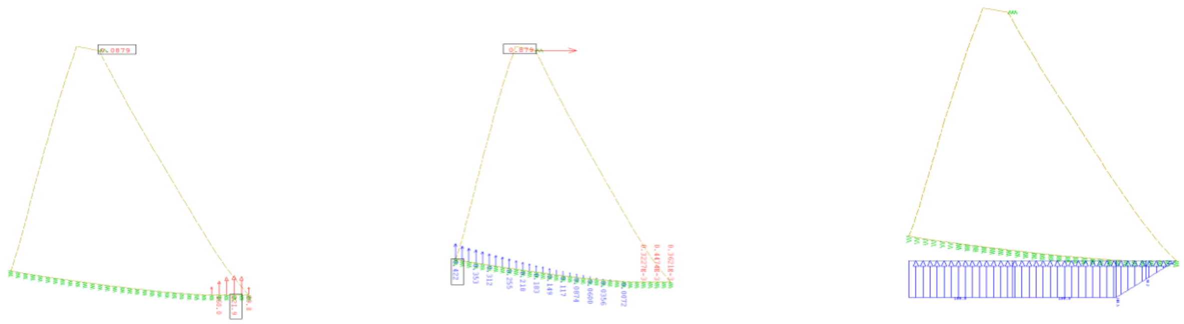

Figure 8: Spring forces (left), deformations (middle) and pore pressure (right) for non-linear analysis. Full pore pressure is assumed in the base area without compression.

The standard load cases, used to determine the capacity according to regulations, has been analyzed with the non-linear model. The results are presented in the table below.

The following table shows the capacity against overturning and sliding. By including cohesion, the capacity against sliding is increased significantly compared to only applying a friction coefficient. The rock bolts contributes only to a small increase in the capacity of 62 kN, and has no effect on the stability. In comparison, the cohesion contributes with approximately 6 000 kN increase in capacity for the standard load cases. Even a small cohesion value of 0.1 MPa could give an increase in capacity of 1 500 kN, compared to a situation when cohesion is not considered.

Table 4: Dam stability (design flood: +0.69 m, PMF: +0.8 m).

		Ice load 100 kN/m	Design flood	PMF	Comment
Overturning (Eccentricity)	Calculated	5.44 m	5.57 m	4.81 m	OK
	Required	5.33 m	5.33 m	2.67 m	
Sliding (Factor of safety, FS)	Only friction	1.36	1.31	1.29	Lower than required SF
	With cohesion	4.31	4.22	4.17	OK
	Required	1.5	1.5	1.1	

Safety criteria for overturning is given by eccentricity of the resultant force from the downstream toe. Safety criteria against sliding is given by Factor of Safety (FS).

The design points that results in failure from the probabilistic analysis has been analyzed in the non-linear model. The non-linear model confirms that the design points are approximately the same using linear springs.

6 Conclusions

The reliability index ranges from 4.88-5.28 for the four design cases. In general, overturning has a higher safety factor than sliding, which corresponds to the results from calculation of the stability in accordance with the traditional method in the dam safety regulations.

The Reinoksvatn dam is classified as a class B dam according to PMCD with a target reliability index above 4.8 with reference period 1 year [1].

The probabilistic analysis show that the dam has a sufficient reliability index within the defined probabilities in chapter 3. As seen by the alpha values in all load cases, the self-weight of the structure is the largest factor of uncertainty in the model. This is uncertainty can be reduced greatly by taking concrete samples and measuring the self-weight. We do not have access to any measurements of the density of the concrete in the dam. The distribution and mean for the self-weight is therefore based on typical values in a design phase. By measuring the concrete weight, the uncertainty can be reduced and the safety index may increase.

The second most significant variable is the water level. The used distributions are roughly estimated based on observations. A more detailed evaluation of the data could give better statistical basis to apply the corresponding flood water level distribution.

The winter season analysis gives highest probability for failure, due to ice load. In reality, the ice load is a deformation load that will disappear even with very small deflections. In addition, a flood water level of 0.5 m above maximum normal operating level (HRV) is possible during the winter season. Restricting this variable can reduce the probability of failure.

The small alpha values for the cohesion, only 1%, may seem surprising. Cohesion is dependent on the area in compression in the failure plane. When, e.g. reducing the self- weight, the area in compression may also reduce. This effect has much greater influence on the results than the value of cohesion itself. This can explain the low alpha value for cohesion.

The alpha value of rock bolts is zero. With a mean value for the yield-strength at 180 MPa and only one ø25 mm bolt per meter along the dam axis, the capacity of these are very small and corresponding contribution to total sliding capacity is negligible.

To increase the reliability of the dam, further investigations of the following factors are recommended:

- **Self-weight:** Carrying out tests of the concrete density could reduce the variance of the probability density function, and lead to better reliability.
- **Ice-pressure measurements**: As the ice load is a deformation load and is in this analysis included as a static load, measurements of the ice-pressure on the dam could give a better indication of the ice load value.
- **Ice-pressure effect:** Implementing the ice load as a small deformation in the analysis. Further investigations and modeling of how the ice pressure affect the dam can also influence the results.
- **Flood levels:** New flood calculations that includes the real statistical distribution of flood events would improve the results. The present flood calculations are based on assumptions regarding the initial water level and do not reflect the actual probability.

7 Acknowledgements

We would like to thank Statkraft and EnergiNorge that has supported this project, and thereby made this probabilistic analysis possible.

8 References

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